CHAPTER 12: INVENTORY MANAGEMENT

Solutions

1. a.

Item	Usage	Unit Cost	Usage x Unit Cost	Category
4021	90	$1,400	$126,000	A
9402	300	12	3,600	C
4066	30	700	21,000	B
6500	150	20	3,000	C
9280	10	1,020	10,200	C
4050	80	140	1,120	C
6850	2,000	10	20,000	B
3010	400	20	8,000	C
4400	5,000	5	25,000	B

In descending order:

Item	Usage x Cost	Category
4021	$126,000	A
4400	25,000	B
4066	21,000	B
6850	20,000	B
9280	10,200	C
3010	8,000	C
9402	3,600	C
6500	3,000	C
4050	1,120	C
	217,920	

1. b.	Category	Percent of Items	Percent of Total Cost
	A	11.1%	57.8%
	B	33.3%	30.2%
	C	55.6%	11.9%

Solutions (continued)

2. The following table contains figures on the monthly volume and unit costs for a random sample of 16 items for a list of 2,000 inventory items.

Item	Unit Cost	Usage	Dollar Usage	Category
K34	10	200	2,000	C
K35	25	600	15,000	A
K36	36	150	5,400	B
M10	16	25	400	C
M20	20	80	1,600	C
Z45	80	250	16,000	A
F14	20	300	6,000	B
F95	30	800	24,000	A
F99	20	60	1,200	C
D45	10	550	5,500	B
D48	12	90	1,080	C
D52	15	110	1,650	C
D57	40	120	4,800	B
N08	30	40	1,200	C
P05	16	500	8,000	B
P09	10	30	300	C

a. Develop an A-B-C classification for these items. [See table.]

b. How could the manager use this information? To allocate control efforts.

c. Suppose after reviewing your classification scheme, the manager decides to place item P05 into the "A" category. What would some possible explanations be for that decision?

It might be important for some reason other than dollar usage, such as cost of a stockout, usage highly correlated to an A item, etc.

3. D = 4,860 bags/yr.

S = $10

H = $75

a. $Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4{,}860)10}{75}} = 36\text{ bags}$

b. Q/2 = 36/2 = 18 bags

c. $\frac{D}{Q} = \frac{4{,}860\text{ bags}}{36\text{ bags / orders}} = 135\text{ orders}$

Solutions (continued)

d. $TC = Q/2H + \frac{D}{Q}S$

$$= \frac{36}{2}(75) + \frac{4{,}860}{36}(10) = 1{,}350 + 1{,}350 = \$2{,}700$$

e. Using S = \$5, Q = $\sqrt{\frac{2(4{,}860)(11)}{75}} = 37.757$

$$TC = \frac{37.757}{2}(75) + \frac{4{,}860}{37.757}(11) = 1{,}415.89 + 1{,}415.90 = \$2{,}831.79$$

Increase by [\$2,831.79 – \$2,700] = \$131.79

4. D = 40/day x 260 days/yr. = 10,400 packages

S = \$60 H = \$30

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(10{,}400)60}{30}} = 203.96 = 204\,\text{boxes}$

b. $TC = \frac{Q}{2}H + \frac{D}{Q}S$

$$= \frac{204}{2}(30) + \frac{10{,}400}{204}(60) = 3{,}060 + 3{,}058.82 = \$6{,}118.82$$

c. Yes

d. $TC_{200} = \frac{200}{2}(30) + \frac{10{,}400}{200}(60)$

TC_{200} = 3,000 + 3,120 = \$6,120

6,120 – 6,118.82 (only \$1.18 higher than with EOQ, so 200 is acceptable.)

5. D = 750 pots/mo. x 12 mo./yr. = 9,000 pots/yr.

Price = \$2/pot S = \$20 P = \$50 H = (\$2)(.30) = \$.60/unit/year

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9{,}000)20}{.60}} = 774.60 \approx 775$

$$TC = \frac{774.6}{2}(.60) + \frac{9{,}000}{774.6}(20)$$

TC = 232.35 + 232.36

= 464.71

If Q = 1500

Solutions (continued)

$$TC = \frac{1{,}500}{2}(.6) + \frac{9{,}000}{1{,}500}(20)$$

TC = 450 + 120 = \$570

Therefore the additional cost of staying with the order size of 1,500 is:

\$570 – \$464.71 = \$105.29

b. Only about one half of the storage space would be needed.

6. u = 800/month, so D = 12(800) = 9,600 crates/yr.

H = .35P = .35(\$10) = \$3.50/crate per yr.

S = \$28

$$\text{Present TC}: \frac{800}{2}(3.50) + \frac{9{,}600}{800}(28) = \$1{,}736$$

a. $$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9{,}600)\$28}{\$3.50}} = 391.93 \text{ [round to } 392]$$

TC at EOQ: $\frac{392}{2}(3.50) + \frac{9{,}600}{392}(28) = \$1{,}371.71.$ Savings approx. \$364.28 per year.

7. H = \$2/month
S = \$55

D_1 = 100/month (months 1–6)
D_2 = 150/month (months 7–12)

a. $$Q_0 = \sqrt{\frac{2DS}{H}} \qquad D_1 : Q_0 = \sqrt{\frac{2(100)55}{2}} = 74.16$$

$$D_2 : Q_0 = \sqrt{\frac{2(150)55}{2}} = 90.83$$

b. The EOQ model requires this.

c. Discount of \$10/order is equivalent to S – 10 = \$45 (revised ordering cost)

1–6 TC_{74} = \$148.32

$$TC_{50} = \frac{50}{2}(2) + \frac{100}{50}(45) = \$140*$$

$$TC_{100} = \frac{100}{2}(2) + \frac{100}{100}(45) = \$145$$

$$TC_{150} = \frac{150}{2}(2) + \frac{100}{150}(45) = \$180$$

Solutions (continued)

7–12 $TC_{91} =$ $181.66

$$TC_{50} = \frac{50}{2}(2) + \frac{150}{50}(45) = \$185$$

$$TC_{100} = \frac{100}{2}(2) + \frac{150}{100}(45) = \$167.5*$$

$$TC_{150} = \frac{150}{2}(2) + \frac{150}{150}(45) = \$195$$

8. D = 27,000 jars/month

H = $.18/month

S = $60

a. $$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(27{,}000)60}{.18}} = 4{,}242.64 \approx 4{,}243.$$

$$TC = \frac{Q}{2}H + \frac{D}{Q}S$$

$TC_{4,000}$ = $765.00

$TC_{4,243}$ = $736.67

$1.32 Difference

$$TC_{4000} = \left(\frac{4{,}000}{2}\right)(.18) + \left(\frac{27{,}000}{4{,}000}\right)(60) = 765$$

$$TC_{4243} = \left(\frac{4{,}243}{2}\right)(.18) + \left(\frac{27{,}000}{4{,}243}\right)60 = 763.68$$

b. Current: $\frac{D}{Q} = \frac{27{,}000}{4{,}000} = 6.75$

For $\frac{D}{Q}$ to equal 10, Q must be 2,700

$$Q = \sqrt{\frac{2DS}{H}} \quad \text{So } 2{,}700 = \sqrt{\frac{2(27{,}000)S}{.18}}$$

Solving, S = $24.30

c. the carrying cost happened to increase rather dramatically from $.18 to approximately $.3705.

$$Q = \sqrt{\frac{2DS}{H}} = 2{,}700 = \sqrt{\frac{2(27{,}000)50}{H}}$$

Solving, H = $.3705

Solutions (continued)

9. $p = 5{,}000$ hotdogs/day

$\left.\begin{array}{l}u = 250 \text{ hotdogs/day}\\ 300 \text{ days per year}\end{array}\right\}$ D= 250/day x 300 days/yr. = 75,000 hotdogs/yr.

S = \$66

H = \$.45/hotdog per yr.

a. $$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(75{,}000)66}{.45}}\sqrt{\frac{5{,}000}{4{,}750}} = 4{,}812.27 \text{ [round to } 4{,}812]$$

b. D/Q_o = 75,000/4,812 = 15.59, or about 16 runs/yr.

c. run length: Q_o/p = 4,812/5,000 = .96 days, or approximately 1 day

10. p = 50/ton/day

$\left.\begin{array}{l}u = 20 \text{ tons/day}\\ 200 \text{ days/yr.}\end{array}\right\}$ D= 20 tons/day x 200 days/yr. = 4,000 tons/yr.

S = \$100

H = \$5/ton per yr.

a. $$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(4{,}000)100}{5}}\sqrt{\frac{50}{50-20}} = 516.40 \text{ tons[10,328 bags]}$$

b. $$I_{max} = \frac{Q}{P}(p-u) = \frac{516.4}{50}(30) = 309.84 \text{ tons[approx 6,196.8 bags]}$$

Average is $\frac{I_{max}}{2} : \frac{309.48}{2} = 154.92$ tons [approx. 3,098 bags]

c. Run length = $\frac{Q}{P} = \frac{516.4}{50} = 10.33$ days

d. Runs per year: $\frac{D}{Q} = \frac{4{,}000}{516.4} = 7.75$ [approx. 8]

e. $Q' = 258.2$

$$TC = \frac{I_{max}}{2}H + \frac{D}{Q}S$$

$TC_{orig.}$ = \$1,549.00

$TC_{rev.}$ = $\underline{\$\ \ 774.50}$

Savings would be \$774.50

Solutions (continued)

11. S = $300

D = 20,000 (250 x 80 = 20,000)

H = $10.00

p = 200/day

u = 80/day

a. $Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(20{,}000)300}{10}}\sqrt{\frac{200}{200-80}} =$

Q_0 = (1,095.451) (1.2910) = 1,414 units

b. Run length = $\frac{Q}{P} = \frac{1{,}414}{200} = 7.07 \text{ days}$

c. 200 – 80 = 120 units per day

d. $I_{max} = \frac{Q}{P}(p-u) = \frac{1{,}414}{200}(200-80) = 848.0 \text{ units}$

848 ÷ 80/day = 10.6 days

<u>- 1.0</u> setup

9.6 days

No, because present demand could not be met.

e. 1) Try to shorten setup time by .40 days.
2) Increase the run quantity of the new product to allow a longer time between runs.
3) Reduce the run size of the other job.]

f. In order to be able to accommodate a job of 10 days, plus one day for setup, there would need to be an11 day supply at Imax, which would be 880 units on hand. Solving the following for Q, we find:

$$I_{max} = \frac{Q}{P}(p-u) = \frac{Q}{200}(200-80) = 880 \text{ units}$$

Q = 1,467.

Using formula 12-4 for total cost, we have

TC @ 1,467 units = $8,489.98
TC @ 1,414 units = <u>$8,483.28</u>
Additional cost = $6.70

12. p = 800 units per day

d = 300 units per day

Q_0 = 2000 units per day

a. Number of batches of heating elements per year = $\frac{75{,}000}{2{,}000} = 37.5$ batches per year

b. The number of units produced in two days = (2 days)(800 units/day) = 1600 units

The number of units used in two days = (2 days) (300 units per day) = 600 units

Current inventory of the heating unit = 0

Inventory build up after the first two days of production = 1,600 – 600 = 1,000 units

Total inventory after the first two days of production = 0 + 1,000 = 1,000 units.

Solutions (continued)

c. Maximum inventory or I_{max} can be found using the following equation:

$$I_{max} = Q_0\left(\frac{p-d}{p}\right) = 2{,}000\left(\frac{800-300}{800}\right) = (2{,}000)(.625) = 1{,}250 \text{ units}$$

$$\text{Average inventory} = \frac{I_{max}}{2} = \frac{1{,}250}{2} = 625 \text{ units}$$

d. Production time per batch = $\frac{Q}{P} = \frac{2{,}000}{800} = 2.5$ days

Setup time per batch = ½ day

Total time per batch = 2.5 + 0.5 = 3 days

Since the time of production for the second component is 4 days, total time required for both components is 7 days (3 + 4). Since we have to make 37.5 batches of the heating element per year, we need (37.5 batches) x (7 days) = 262.5 days per year.

262.5 days exceed the number of working days of 250, therefore we can conclude that there is not sufficient time to do the new component (job) between production of batches of heating elements.

An alternative approach for part d is:

The max inventory of 1,250 will last 1250/300 = 4.17 days

4.17 – .50 day for setup = 3.67 days. Since 3.67 is less than 4 days, there is not enough time.

13. D = 18,000 boxes/yr.

S = $96

H = $.60/box per yr.

a. $Q_o = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(18{,}000)96}{.60}} = 2{,}400$ boxes

Since this quantity is feasible in the range 2000 to 4,999, its total cost and the total cost of all lower price breaks (i.e., 5,000 and 10,000) must be compared to see which is lowest.

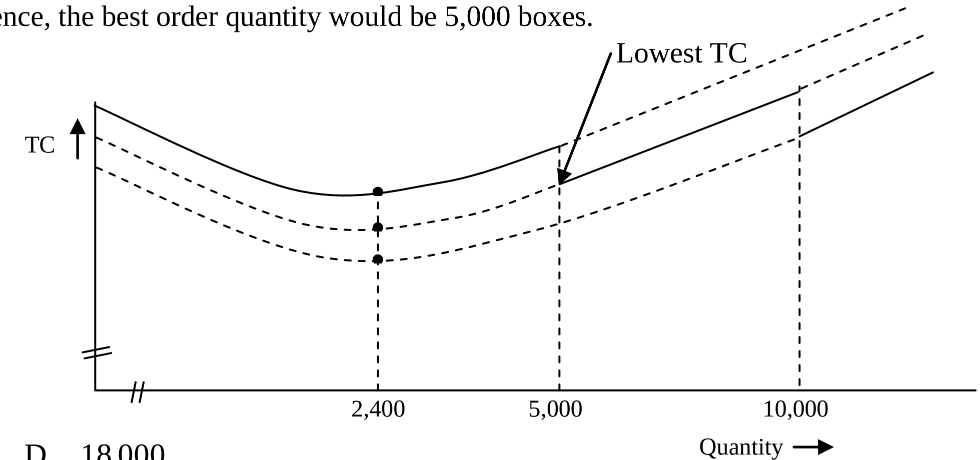
$$TC_{2,400} = \frac{2,400}{2}(.60) + \frac{18,000}{2,400}(\$96) + \$1.20(18,000) = \$23,040$$

$$TC_{5,000} = \frac{5,000}{2}(.60) + \frac{18,000}{5,000}(\$96) + \$1.15(18,000) = \$22,545.6\,[\text{lowest}]$$

$$TC_{10,000} = \frac{10,000}{2}(.60) + \frac{18,000}{10,000}(\$96) + \$1.10(18,000) = \$22,972.80$$

Solutions (continued)

Hence, the best order quantity would be 5,000 boxes.



b. $\frac{D}{Q} = \frac{18,000}{5,000} = 3.6$ orders per year

14. a. S = $48

D = 25 stones/day x 200 days/yr. = 5,000 stones/yr.

Quantity	Unit Price
1 – 399	$10
400 – 599	9
600 +	8

a. H = $2

$$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(5,000)48}{2}} = 489.90$$

$$TC_{490} = \frac{490}{2}2 + \frac{5,000}{490}48 + 9\ (5,000) = \$45,980$$

$$TC_{600} = \frac{600}{2}2 + \frac{5,000}{600}48 + 8\ (5,000) = \$41,000$$

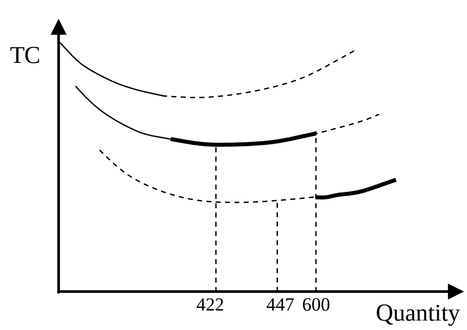
$\therefore$ 600 is optimum.

b. H = .30P

$$EOQ_{\$8} = \sqrt{\frac{2(5,000)48}{.30(8)}} = 447\ \text{NF}$$

(Not feasible at $8/stone)

$$EOQ_{\$9} = \sqrt{\frac{2(5,000)48}{.30(9)}} = 422$$



Solutions (continued)

(Feasible)

Compare total costs of the EOQ at \$9 and lower curve's price break:

$$TC = \frac{Q}{2}(.30P) + \frac{D}{Q}(S) + PD$$

$$TC_{422} = \frac{422}{2}[.30(\$9)] + \frac{5{,}000}{422}(\$48) + \$9(5{,}000) = \$46{,}139$$

$$TC_{600} = \frac{600}{2}[.30(\$8)] + \frac{5{,}000}{600}(\$48) + \$8(5{,}000) = \$41{,}120$$

Since an order quantity of 600 would have a lower cost than 422, 600 stones is the optimum order size.

c. ROP = 25 stones/day (6 days) = 150 stones.

15.

	Range	P	H	Q
D = 4,900 seats/yr.	0–999	\$5.00	\$2.00	495
H = .4P	1,000–3,999	4.95	1.98	497 NF
S = \$50	4,000–5,999	4.90	1.96	500 NF
	6,000+	4.85	1.94	503 NF

Compare TC_{495} with TC for all lower price breaks:

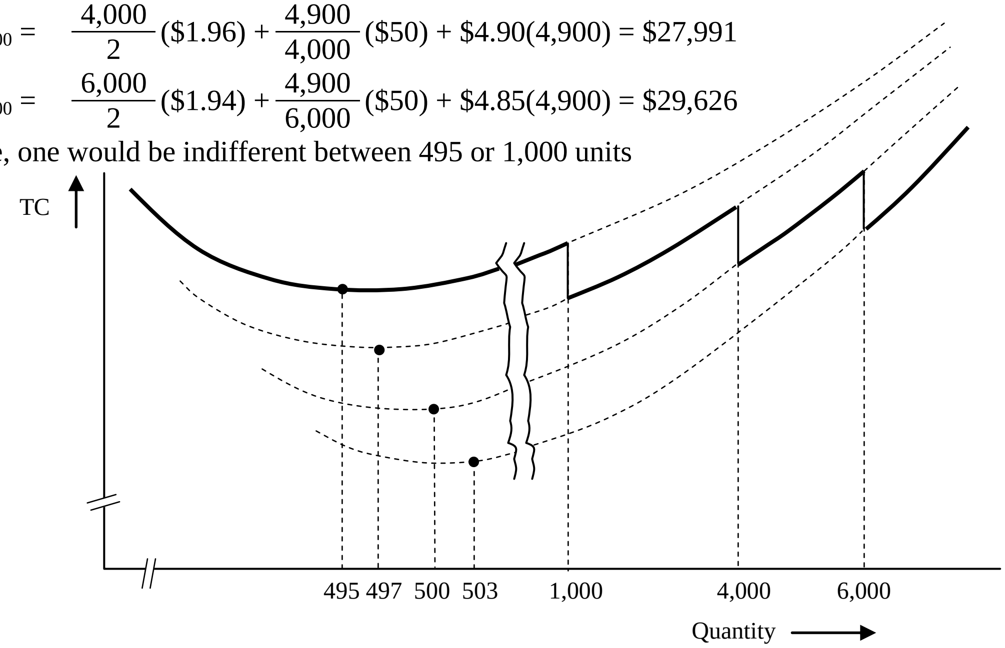
$$TC_{495} = \frac{495}{2}(\$2) + \frac{4{,}900}{495}(\$50) + \$5.00(4{,}900) = \$25{,}490$$

$$TC_{1{,}000} = \frac{1{,}000}{2}(\$1.98) + \frac{4{,}900}{1{,}000}(\$50) + \$4.95(4{,}900) = \$25{,}490$$

$$TC_{4{,}000} = \frac{4{,}000}{2}(\$1.96) + \frac{4{,}900}{4{,}000}(\$50) + \$4.90(4{,}900) = \$27{,}991$$

$$TC_{6{,}000} = \frac{6{,}000}{2}(\$1.94) + \frac{4{,}900}{6{,}000}(\$50) + \$4.85(4{,}900) = \$29{,}626$$

Hence, one would be indifferent between 495 or 1,000 units



Solutions (continued)

16. D = (800) x (12) = 9600 units
S = $40
H = (25%) x P

For Supplier A:

$$Q_{13.6} = \sqrt{\frac{2(9,600)(40)}{(.25)(13.6)}} = 475.27\,(not\ \ feasible\,)$$

$$Q_{13.8} = \sqrt{\frac{2(9,600)(40)}{(.25)(13.8)}} = 471.81$$

$$TC_{471.81} = \left(\frac{471.81}{2}\right)3.45 + \frac{9,600}{471.81}(40) + [(13.8)(9,600)]$$

$$TC_{471.81} = 813.88 + 813.87 + 132,480$$

$$TC_{471.81} = \$134,107.75$$

$$TC_{500} = \left(\frac{500}{2}\right)(.25)(13.6) + \frac{9,600}{500}(40) + [(13.6)(9,600)]$$

$$TC_{500} = 768 + 850 + 130,560$$

$$TC_{500} = \$132,178\,*$$

For Supplier B:

$$Q_{13.7} = \sqrt{\frac{2(9,600)(40)}{(.25)(13.7)}} = 473.53$$

$$TC_{471.81} = \left(\frac{473.53}{2}\right)(.25)(13.7) + \frac{9,600}{473.53}(40) + [(13.7)(9\quad 0)]$$

$$TC_{471.81} = 810.93 + 810.92 + 131,520$$

$$TC_{471.81} = \$133,141.85$$

Since $132,178 < $133,141.85, choose supplier A.
The optimal order quantity is 500 units.

Solution (continued)

17. D = 3600 boxes per year
Q = 800 boxes (recommended)
S = $80 /order
H = $10 /order

If the firm decides to order 800, the total cost is computed as follows:

$$TC = \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + (P * D)$$

$$TC_{Q=800} = \left(\frac{800}{2}\right)\$10 + \left(\frac{3{,}600}{800}\right)\$80 + (3{,}600 \text{ x } 1.1)$$

$$TC_{Q=800} = 4{,}000 + 360 + 3{,}960 = 8{,}320$$

Even though the inventory total cost curve is fairly flat around its minimum, when there are quantity discounts, there are multiple U shaped total inventory cost curves for each unit price depending on the unit price. Therefore when the quantity changes from 800 to 801, we shift to a different total cost curve.

If we take advantage of the quantity discount and order 801 units, the total cost is computed as follows:

$$TC = \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + (P * D)$$

$$TC_{Q=801} = \left(\frac{801}{2}\right)\$10 + \left(\frac{3{,}600}{801}\right)\$80 + (3{,}600 \text{ x } 1.0)$$

$$TC_{Q=801} = 4{,}005 + 359.55 + 3{,}600 = 7{,}964.55$$

The order quantity of 801 is preferred to order quantity of 800 because $TC_{Q=801} < TC_{Q=800}$ or 7964.55 < 8320.

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3{,}600)(80)}{10}} = 240 \text{ boxes}$$

$$TC_{EOQ} = \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + (P * D)$$

$$TC_{EOQ} = \left(\frac{240}{2}\right)\$10 + \left(\frac{3{,}600}{240}\right)\$80 + (3{,}600 \text{ x } 1.1)$$

$$TC_{EOQ} = 1{,}200 + 1{,}200 + 3{,}960 = 6{,}360$$

The order quantity of 800 is not around the flat portion of the curve because the optimal order quantity (EOQ) is much lower than the suggested order quantity of 800. Since the EOQ of 240 boxes provides the lowest total cost, it is the recommended order size.

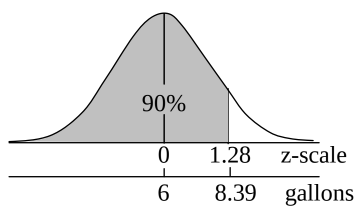
Solution (continued)

18. Daily usage = 800 ft./day Lead time = 6 days
Service level desired: 95 percent. Hence, risk should be 1.00 – .95 = .05
This requires a safety stock of 1,800 feet.
ROP = expected usage + safety stock
= 800 ft./day x 6 days + 1,800 ft. = 6,600 ft.

19. expected demand during LT = 300 units
σ_{dLT} = 30 units
a. Z = 2.33, ROP = exp. demand + $Z\sigma_{d\,LT}$
300 + 2.33 (30) = 369.9 ∏ 370 units
b. 70 units (from a.)
c. smaller Z ∏ less safety stock
ROP smaller:

20. LT demand = 600 lb.
$\sigma_{d\,LT}$ = 52 lb.
risk = 4% ∏ Z = 1.75
a. ss = $Z\sigma_{d\,LT}$ = 1.75 (52 lbs.) = 91 lbs.
b. ROP = Average demand during lead time + safety stock
ROP = 600 + 91 = 691 lbs.
c. With no safety stock risk is 50%.

21. $\bar{d}$ = 21 gal./wk.
σ_d = 3.5 gal./wk.
LT = 2 days
SL = 90 percent requires z = +1.28



a. $\text{ROP} = \bar{d}(\text{LT}) + z\sqrt{\text{LT}}(\sigma_d) = 21(2/7) + 1.28\sqrt{(2/7)}(3.5) = 8.39$ gallons

b. $Q = \bar{d}(\text{OI} + \text{LT}) + Z\sigma_d\sqrt{\text{OI} + \text{LT}} - A = 21\left(\frac{10}{7} + \frac{2}{7}\right) + 1.28(3.5)\sqrt{12/7} - 8 = 33.87$

or approx. 34 gal./wk.

Average demand per day = 21 gallons / 7days per week = 3 gallons
μ = Average demand during lead time = (3 gallons) (2 days) = 6 gallons

$$\sigma_L = \sqrt{\frac{2}{7}}(3.5) = 1.871$$

$$Z = \frac{\text{ROP} - \mu}{\sigma_L} = \frac{8 - 6}{1.871} = 1.069$$

Z is approximately 1.07. From Appendix B, Table B, the lead time service level is .8577.

Solution (continued)

c. 1 day after From a, ROP = 8.39

2 more days

on hand = ROP – 2 gal. = 6.39 $6.39 = 21\,(2/7) + Z\sqrt{2/7}\;(3.5)$

P (stockout)= ? solving, $Z = \dfrac{6.39-6}{1.871} = .208 \approx .21$

d = 21 gal./wk. From Appendix B, Table B, Z=.21 gives a risk of

σ_d = 3.5 gal./wk. 1 – .5832 = .4168 or about 42%

22. d = 30 gal./day
ROP = 170 gal.
LT = 4 days ss = $Z\sigma_{d\,LT}$ = 50
ss = 50 gal.
Risk = 9% Z = 1.34 Solving, $\sigma_{d\,LT}$ = 37.31
3% ∏ Z = 1.88 x 37.31 = 70.14 gal.

23.

D	= 85 boards/day	ROP	$= d \times \overline{LT} + Z\, d\, \sigma_{LT}$
ROP	= 625 boards	625	= 85 x 6 + Z (85) 1.1
$\overline{LT}$	= 6 days	Z	= 1.23 → 10.93%
σ_{LT}	= 1.1 day		.1093 approx. 11%

24. SL ≥ 96% → Z = 1.75

$\bar{d}$ = 12 units/day $\overline{LT}$ = 4 days

σ_d = 2 units/day σ_{LT} = 1 day

$$ROP = \overline{d}\,\overline{LT} + Z\sqrt{\overline{LT}\sigma_d^{\,2} + \bar{d}^{\,2}\sigma_{LT}^{\,2}}$$

$$= 12\,(4) + 1.75\sqrt{4(4) + 144(1)}$$

= 48 + 1.75 (12.65)
= 48 + 22.14
= 70.14

Solutions (continued)

25. LT = 3 days S = $30
D = 4,500 gal H = $3
360 days/yr.

$$\bar{d} = \frac{4{,}500}{360} = 12.5/day$$

σ_d = 2 gal.
Risk = 1.5% → Z = 2.17

a.

Qty.	Unit Price
1 – 399	$2.00
400 – 799	1.70
800+	1.62

$Q_o = \sqrt{\dfrac{2DS}{H}} = 300$

TC = Q/2 H + D/Q S + PD

TC_{300} = 150 (3) + 15 (30) + 2(4,500) = $9,900

TC_{400} = 200(3) + 11.25(30) + 1.70(4,500) = \$8,587.50*

TC_{800} = 400(3) + 5.625(30) + 1.62(4,500) = \$8,658.75

b. ROP $= \bar{d}\,LT + Z\sqrt{LT}\sigma_d$

$= 12.5\,(3) + 2.17\,\sqrt{3}\,(2)$

$= 37.5 + 7.517$

$= 45.017$ gal.

26. d = 5 boxes/wk.

σ_d = .5 boxes/wk.

LT = 2 wk.

S = \$2

H = \$.20/box

a. D = .5 boxes/wk. x 52 wk./yr. = 26 boxes/yr.

$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(260)2}{.20}} = 72.11\,[\text{round to } 72]$$

b. ROP $= \bar{d}\,(LT) + z\,\sqrt{LT}\,(\sigma_d)$

$$z = \frac{ROP - \bar{d}(LT)}{\sqrt{LT}(\sigma_d)} = \frac{12 - 5(2)}{\sqrt{2}(.5)} = 2.83$$

Area under curve to left is .9977, so risk = 1.0000 – .9977 = .0023

Solutions (continued)

c. $Q_0 = \bar{d}(OI + LT) + z\sigma_d\sqrt{OI + LT} - A$

Thus,

$36 = 5(7 + 2) + z(.5)\sqrt{7+2} - 12$

Solving for z yields z = +2.00 which implies a risk of 1.000 – .9772 = .0228.

27. $\bar{d}$ = 80 lb.

σ_d = 10 lb.

$\overline{LT}$ = 8 days

σ_{LT} = 1 day

SL = 90 percent, so z = +1.28

a. ROP = $\bar{d}\,(\overline{LT}) + z\sqrt{\overline{LT}\sigma^2_d + \bar{d}^2\sigma^2_{LT}}$

$= 80\,(8) + 1.28\sqrt{8(10)^2 + 80^2(1)^2} = 640 + 1.28\,(84.85)$

= 748.61 [round to 749]

b. $E(n) = E(z)\,d_{LT} = .048(84.85) = 4.073$ units

28. D = 10 rolls/day x 360 days/yr. = 3,600 rolls/yr.

$\bar{d}$ = 10 rolls/day LT = 3 days H = $.40/roll per yr.

σ_d = 2 rolls/day S = $1

a. $Q_0 = \sqrt{\dfrac{2DS}{H}} = \sqrt{\dfrac{2(3,600)1}{.40}} = 134.16$ [round to 134]

b. SL of 96 percent requires z = +1.75

ROP = $\bar{d}\,(LT) + z\sqrt{LT}\,(\sigma_d) = 10(3) + 1.75\sqrt{(3)}\,(2) = 36.06$ [round to 36]

c. $E(n) = E(z)\,\sigma_{d\,LT} = .016(\sqrt{LT})(\sigma_d) = .016\,\sqrt{3}\,(2) = .0554$/cycle

$E(N) = E(n)\,\dfrac{D}{Q} = .0554\,\dfrac{3600}{134} = 1.488$ or about 1.5 rolls

d. $1 - SL_{annual} = E(z)\dfrac{\sigma_{dLT}}{Q} = (.016)\dfrac{3.464}{134.16} = .000413$

$SL_{annual} = 1 - .000413 = .9996$

Solutions (continued)

29. (Partial Solution)

$SL_{annual} = 99\%$

a. SS = ? b. risk = ?

$dLT = 80$, $\sigma_{d\,LT} = 5$

$Q_o = 179$ cases

$$SL_{annual} = 1 - \frac{E(z)\,\sigma_{d\,LT}}{Q}$$

$$.99 = 1 - \frac{E(z)\,\sigma_{d\,LT}}{179}$$

Solving, E(z) = 0.358
From Table 12–3, Z = 0.08
Hence, the probability of a stockout is 1-.8577=.1423.

a. $ss = Z\sigma_{d\,LT}$
$= .08\,(5) = .40$ cases

b. $1 - .5319 = .4681$

30. $\bar{d} = 250$ gal./wk. H = \$1/month D = 250 gal./wk. x 52 wk./yr. = 13,000 gal./yr.

$\sigma_d = 14$ gal./wk. S = \$20

LT = 10 wk. .5 Yes

$SL_{annual} = 98\%$

$$Q_0 = \sqrt{\frac{2DS}{H}} = \frac{2(13{,}000)20}{12} = 208.16 \approx 208$$

a. $$SL_{annual} = 1 - \frac{\sum(z)\,\sigma_{dLT}}{Q}$$

$\sqrt{\sigma_{dLT}} = LT\,\sigma_d$

$= \sqrt{5}\,(14)$

$= 9.90$

$$.02 = \frac{\sum(z)\,9.90}{208}$$

$E(z) = .42 \rightarrow z = -.04$ $SS = -.04(9.90) = -.40$

$SL = .4840$

b. $E(n) = E(z)\,\sigma_{dLT}$

$5 = E(z)9.90$

$E(z) = .505 \rightarrow z = -.20$ $SS = z\sigma_{dLT}$

$SL = .4207$ $SS = -.20(9.90) = -1.98$ units

31. FOI

$Q = \bar{d}\,(OI + LT) + z\sigma_d\sqrt{OT + LT} - A$

$= \bar{d}\,(16) + 2.05\sigma_d\,\sqrt{16} - A$

SL = .98

OI = 14 days

LT = 2 days

D = 40/day

d = 40/day

$\sigma_2 = 3$/day

Cycle	
1	$640 + 2.05(3)\sqrt{16} - 42 = 622.6 \rightarrow 623$ units
2	$640 + 2.05(3)\sqrt{16} - 8 = 656.6 \rightarrow 657$ units
3	$640 + 2.05(3)\sqrt{16} - 103 = 561.6 \rightarrow 562$ units

Solutions (continued)

32. 50 wk./yr.

P34	P35
$D = 3{,}000$ units	$D = 3{,}500$ units
$\bar{d} = 60$ units/wk.	$\bar{d} = 70$ units/wk.
$\sigma_d = 4$ units/wk.	$\sigma_d = 5$ units/wk.
LT = 2 wk.	LT = 2 wk.
unit cost = \$15	unit cost = \$20
H = (.30)(15) = \$4.50	H = (.30)(20) = 6.00
S = \$70	S = \$30
Risk = 2.5%	Risk = 2.5%

$$Q_{P34}\sqrt{\frac{2(3{,}000)70}{4.50}} = 305.5 \approx 306 \text{ units}$$

$$ROP_{P34} = \bar{d} \text{ x } LT + z\sqrt{LT}\sigma_d$$

$$ROP_{P34} = 60(2) + 1.96\sqrt{2}\,(4) = 131.1$$

$$Q = (OI + LT)\ \bar{d} + z\ \sqrt{LT}\ \sigma_d - A$$

$$Q_{P35} = 70\,(4 + 2) + 1.96\ \sqrt{4+2}\ (5) - 110$$

$$Q_{P35} = 420 + 24 - 110$$

$$Q_{P35} = 334 \text{ units}$$

33. a.

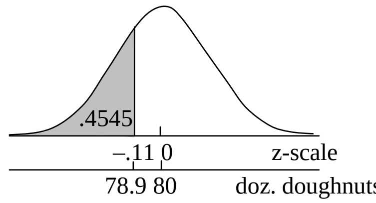
Item	Annual \$ volume	Classification
H4-010	50,000	C
H5–201	240,800	B
P6-400	279,300	B
P6-401	174,000	B
P7-100	56,250	C
P9-103	165,000	C
TS-300	945,000	A
TS-400	1,800,000	A
TS-041	16,000	C
V1-001	132,400	C

b.

Item	Estimated annual demand	Ordering cost	Unit holding cost (\$)	EOQ
H4-010	20,000	50	.50	2,000
H5-201	60,200	60	.80	3,005
P6-400	9,800	80	8.55	428
P6-401	14,500	50	3.60	635
P7-100	6,250	50	2.70	481
P9-103	7,500	50	8.80	292
TS-300	21,000	40	11.25	386
TS-400	45,000	40	10.00	600
TS-041	800	40	5.00	113
V1-001	33,100	25	1.40	1,087

Solutions (continued)

34.



C_s = Rev – Cost = \$4.80 – \$3.20 = \$1.60

C_e = Cost – Salvage = \$3.20 – \$2.40 = \$.80

$$SL = \frac{C_s}{C_s + C_e} = \frac{\$1.60}{\$1.60 + \$.80} = \frac{1.6}{2.4} = .67$$

Since this falls between the cumulative probabilities of .63(x = 24) and .73(x = 25), this means Don should stock 25 dozen doughnuts.

x Demand	P(x)	Cum. P(x)
19	.01	.01
20	.05	.06
21	.12	.18
22	.18	.36
23	.13	.49
24	.14	.63
25	.10	.73
26	.11	.84
27	.10	.94
.	.	.
.	.	.
.	.	.

35. C_s = \$88,000

C_e = \$100 + 1.45(\$100) = \$245

a. $$SL = \frac{C_s}{C_s + C_e} = \frac{\$88{,}000}{\$88{,}000 + \$245} = .9972$$

Using the Poisson probabilities, the minimum level stocking level that will provide the desired service is nine spares (cumulative probability = .998).

[From Poisson Table with $\mu = 3.2$]

x	Cum. Prob.
0	.041
1	.171
2	.380
3	.603
4	.781
5	.895
6	.955
7	.983
8	.994
9	.998
.	.
.	.
.	.

Solutions (continued)

b. $$SL = \frac{C_s}{C_s + C_e}$$

$$.041 = \frac{C_s}{C_s + 245}$$

$$.041(C_s + 245) = C_s$$

$$.041C_s + 10.045 = C_s$$

$$.959C_s = 10.045$$

$$C_s = \$10.47$$

Carrying no spare parts is the best strategy if the shortage cost is less than or equal to \$10.47 ($C_s \le 10.47$).

36. C_s = Rev – Cost = \$5.70 – \$4.20 = \$1.50/unit

C_e = Cost – Salvage = \$4.20 – \$2.40 = \$1.80/unit

$\bar{d}$ = 80 lb./day
σ_d = 10 lb./day

$$SL = \frac{C_s}{C_s + C_e} = \frac{\$1.50}{\$1.50 + \$1.80} = \frac{\$1.50}{\$3.30} = .4545$$

The corresponding z = –.11

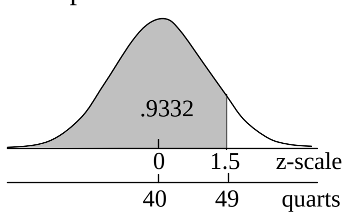
$S_o = \bar{d} - z\,\sigma_d = 80 - .11(10) = 78.9$ lb.

37. $\bar{d}$ = 40 qt./day
d = 6 qt./day
C_e = \$.35/qt
C_s = ?
S = 49 qt.

A stocking level of 49 quarts translates into a z of + 1.5:

$$z = \frac{S - \bar{d}}{\sigma_d} = \frac{49 - 40}{6} = 1.5$$

This implies a service level of .9332:



$$SL = \frac{C_s}{C_s + C_e} \quad \text{Thus, } .9332 = \frac{C_s}{C_s + \$.35}$$

Solving for C_s we find: $.9332(C_s + .35) = C_s$; C_s = \$4.89/qt.

Customers may buy other items along with the strawberries (ice cream, whipped cream, etc.) that they wouldn't buy without the berries.

Solutions (continued)

38. C_s = Rev – Cost = \$12 – \$9 = \$3.00/cake

C_e = Cost – Salvage = \$9 – ½ (\$9.00) = \$4.50/cake

Demand is Poisson with mean of 6

$$SL = \frac{C_s}{C_s + C_e} = \frac{\$3.00}{\$3.00 + \$4.50} = .40$$

Since .40 falls between the cumulative probability for demand of 4 and 5, the optimum stocking level is 5 cakes.

[From Poisson Table with μ = 6.0]

Demand	Cum. Prob.
0	.003
1	.017
2	.062
3	.151
4	.285 ← .40
5	.446
6	.606
.	.
.	.
.	.

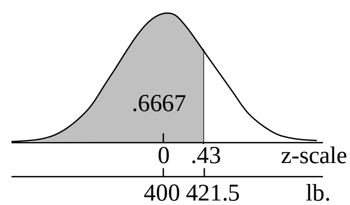
39. C_s = \$.10/burger x 4 burgers/lb. = \$.40/lb.

C_e = Cost – Salvage = \$1.00 – \$.80 = \$.20/lb.

$$SL = \frac{C_s}{C_s + C_e} = \frac{\$.40}{\$.40 + \$.20} = .6667.$$

The appropriate z is +.43.

$S_o = \mu + z\sigma$ = 400 + .43(50) = 421.5 lb.



40. C_s = \$10/machine

C_e = ?

S = 4 machines

Demand	Freq.	Cum. Freq.
0	.30	.30
1	.20	.50
2	.20	.70
3	.15	.85
4	.10	.95
5	.05	1.00
	1.00	

a. For four machines to be optimal, the SL ratio must be $.85 \le \dfrac{\$10}{\$10 + C_e} \le .95$.

Setting the ratio equal to .85 and solving for C_e yields \$1.76, which is the upper end of the range. Setting the ratio equal to .95 and again solving for C_e , we find C_e = \$.53, which is the lower end of the range.

b. The number of machines should be <u>decreased</u>: the higher excess costs are, the lower SL becomes, and hence, the lower the optimum stocking level.

c. For four machines to be optimal, the SL ratio must be between .85 and .95, as in part <u>a</u>. Setting the ratio equal to .85 yields the <u>lower</u> limit:

$$.85 = \frac{C_s}{C_s + \$10}$$ Solving for C_s we find C_s = \$56.67.

Setting the ratio equal to .95 yields the <u>upper</u> end of the range:

$$.95 = \frac{C_s}{C_s + \$10}$$ Solving for C_s we find C_s = \$190.00

Solutions (continued)

41.

# of spares	Probability of Demand	Cumulative Probability
0	0.10	0.10
1	0.50	0.60
2	0.25	0.85
3	0.15	1.00

C_s = Cost of stockout = ($500 per day) (2 days) = $1000
C_e = Cost of excess inventory = Unit cost – Salvage Value = $200 – $50 = $150

$$SL = \frac{C_s}{C_s + C_e} = \frac{1{,}000}{1{,}000 + 150} = .869$$

Since 86.9% is between cumulative probabilities of 85% and 100%, we need to order 3 spares.

42. Demand and the probabilities for the cases of wedding cakes are given in the following table.

Demand	Probability of Demand	Cumulative Probability
0	0.15	0.15
1	0.35	0.50
2	0.30	0.80
3	0.20	1.00

C_s = Cost of stockout = Selling Price – Unit Cost = $60 – $33 = $27
C_e = Cost of excess inventory = Unit Cost – Salvage Value = $33 – $10 = $23

$$SL = \frac{C_s}{C_s + C_e} = \frac{27}{27 + 23} = .54$$

Since the service level of 54% falls between cumulative probabilities of 50% and 80%, the supermarket should stock 2 cases of wedding cakes.

43. Cs = $99, Ce = $200

$$\text{SL} = \frac{99}{99 + 200} = 0.3311. \quad z = -0.44.$$

Overbook: 18 – 0.44(4.55) = 15.998, or 16 tickets.

44. Mean usage = 4.6 units/day
Standard dev. = 1.265 units/day
LT = 3 days
ROP = 18

Using equation 12-13: $18 = 4.6(3) + z(1.265)\sqrt{3}$

Solving, z = 1.92 which gives a service level of 97.26%.